Problem 10.29

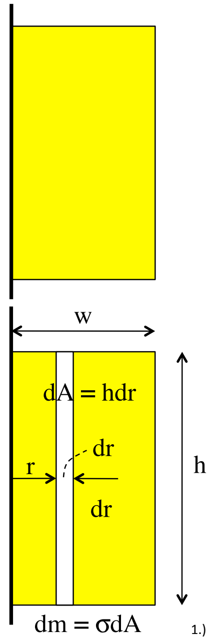
A door of known height, width and mass is hinged along the line shown.

a.) What is its moment of inertia about the line?

The moment of inertia relationship is shown below.

$$I = \int r^2\,dm$$

Much like the *discrete* relationship, it asks you to determine the differential *moment of* inertia “dI” for some mass differential bit of mass “dm” a distance “r” units from the axis in question, then execute the integrate to determine the total moment of inertia. In this case, the easiest way to do it is to define an *area mass density function* σ and notice that any differential area “dA” times that function will give you the differential mass “dm” associated with that area. Note that the differential area of the differential strip is “dA” times the height “h.” The sketch to the right more or less shows it all.

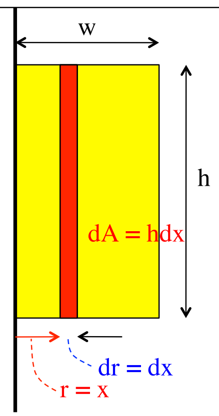


The mass density function can be determined using the total mass “M” and total area “A” of the door, where A equals the width “w” times height “h”, or:

$$\sigma = \frac{M}{wh}$$

With that, we can write:

$$
\begin{aligned}
I &= \int r^2\,dm \\
&= \int_{r=0}^{w} r^2 (\sigma dA) \\
&= \int_{x=0}^{w} x^2 \left(\left(\frac{M}{w\cancel{h}} \right) (\cancel{h}dx) \right) \\
&= \frac{M}{w} \int_{x=0}^{w} x^2\,dx \\
&= \frac{M}{w} \left(\frac{x^3}{3} \right) \Big|_{x=0}^{w} \\
&= \frac{M}{w} \left(\frac{w^3}{3} \right) \\
&= \frac{1}{3} M w^2 \\
&= \frac{1}{3} (23.0\ \text{kg})(.870\ \text{m})^2 \\
&= 5.80\ \text{kg} \bullet \text{m}^2
\end{aligned}
$$



$$\sigma = \frac{dm}{dA} \left(= \frac{M}{A} \right)$$

$$\Rightarrow \quad dm = \sigma dA$$

b.) Any parameters not needed?

As the final expression does not have the door height “h” in it (even though we did use that parameter in the derivation), apparently the door height is not needed to do this calculation.

Note: This shouldn’t be too hard to believe. The height is part of the differential area expression which gets multiplied by the *mass per unit area* function. If the height had been bigger, the mass per unit area would have been smaller and the product would not have changed. The height really didn’t matter.